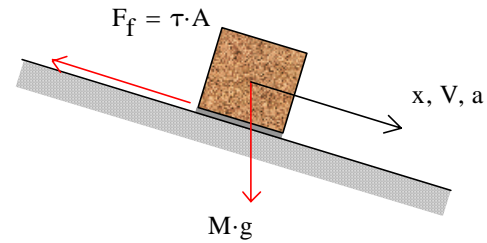


Problem 2.51

[Difficulty: 4]

2.51 A block 0.1 m square, with 5 kg mass, slides down a smooth incline, 30° below the horizontal, on a film of SAE 30 oil at 20°C that is 0.20 mm thick. If the block is released from rest at $t=0$, what is its initial acceleration? Derive an expression for the speed of the block as a function of time. Plot the curve for $V(t)$. Find the speed after 0.1 s. If we want the mass to instead reach a speed of 0.3 m/s at this time, find the viscosity μ of the oil we would have to use.



Given: Data on the block and incline

Find: Initial acceleration; formula for speed of block; plot; find speed after 0.1 s. Find oil viscosity if speed is 0.3 m/s after 0.1 s

Solution:

Given data $M = 5 \cdot \text{kg}$ $A = (0.1 \cdot \text{m})^2$ $d = 0.2 \cdot \text{mm}$ $\theta = 30 \cdot \text{deg}$

From Fig. A.2 $\mu = 0.4 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$

Applying Newton's 2nd law to initial instant (no friction) $M \cdot a = M \cdot g \cdot \sin(\theta) - F_f = M \cdot g \cdot \sin(\theta)$

$$\text{so } a_{\text{init}} = g \cdot \sin(\theta) = 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times \sin(30 \cdot \text{deg}) \quad a_{\text{init}} = 4.9 \frac{\text{m}}{\text{s}^2}$$

Applying Newton's 2nd law at any instant $M \cdot a = M \cdot g \cdot \sin(\theta) - F_f$ and $F_f = \tau \cdot A = \mu \cdot \frac{du}{dy} \cdot A = \mu \cdot \frac{V}{d} \cdot A$

$$\text{so } M \cdot a = M \cdot \frac{dV}{dt} = M \cdot g \cdot \sin(\theta) - \frac{\mu \cdot A}{d} \cdot V$$

Separating variables

$$\frac{dV}{g \cdot \sin(\theta) - \frac{\mu \cdot A}{M \cdot d} \cdot V} = dt$$

Integrating and using limits

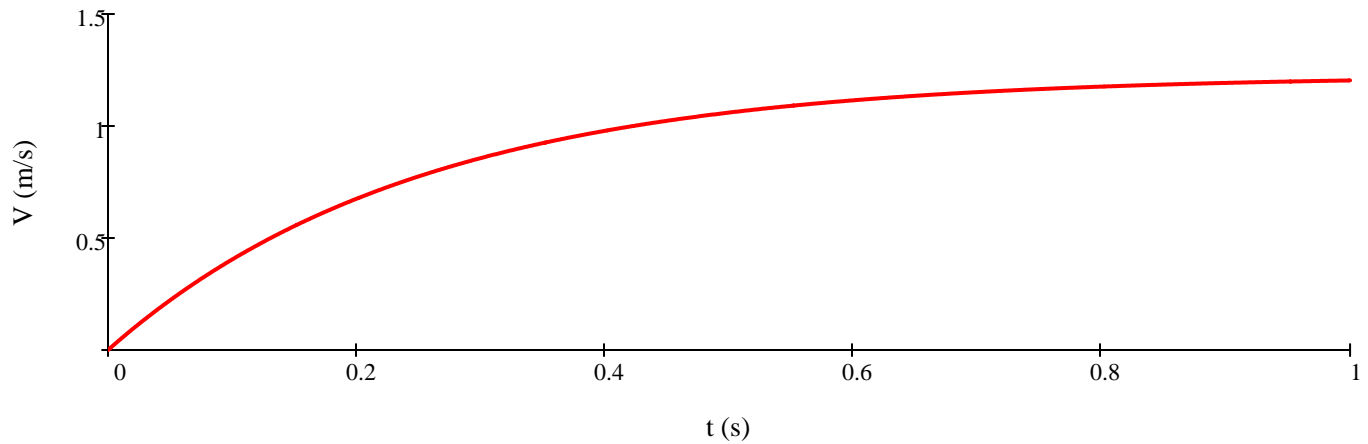
$$-\frac{M \cdot d}{\mu \cdot A} \cdot \ln \left(1 - \frac{\mu \cdot A}{M \cdot g \cdot d \cdot \sin(\theta)} \cdot V \right) = t$$

$$\text{or } V(t) = \frac{M \cdot g \cdot d \cdot \sin(\theta)}{\mu \cdot A} \cdot \left(1 - e^{\frac{-\mu \cdot A}{M \cdot d} \cdot t} \right)$$

$$\text{At } t = 0.1 \text{ s} \quad V = 5 \cdot \text{kg} \times 9.81 \cdot \frac{\text{m}}{\text{s}^2} \times 0.0002 \cdot \text{m} \cdot \sin(30 \cdot \text{deg}) \times \frac{\text{m}^2}{0.4 \cdot \text{N} \cdot \text{s} \cdot (0.1 \cdot \text{m})^2} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \times \left[1 - e^{\left(\frac{0.4 \cdot 0.01}{5 \cdot 0.0002} \cdot 0.1 \right)} \right]$$

$$V(0.1 \cdot \text{s}) = 0.404 \cdot \frac{\text{m}}{\text{s}}$$

The plot looks like



To find the viscosity for which $V(0.1 \text{ s}) = 0.3 \text{ m/s}$, we must solve

$$V(t = 0.1 \cdot \text{s}) = \frac{M \cdot g \cdot d \cdot \sin(\theta)}{\mu \cdot A} \cdot \left[1 - e^{\frac{-\mu \cdot A}{M \cdot d} \cdot (t=0.1 \cdot \text{s})} \right]$$

The viscosity μ is implicit in this equation, so solution must be found by manual iteration, or by any of a number of classic root-finding numerical methods, or by using *Excel's Goal Seek*

Using *Excel*:

$$\mu = 1.08 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$